

PHYSICS 225A - SET 1

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(a) $M_W = 80.425 \pm 0.038 \text{ GeV}/c^2$
 $M_Z = 91.1876 \pm 0.0021 \text{ GeV}/c^2$
 $M_p = 0.93827200 \pm 0.00000004 \text{ GeV}/c^2$

These values may not be the latest/greatest

$$\Delta E \Delta t \sim \hbar \Rightarrow R \sim c \Delta t \sim \frac{\hbar c}{\Delta E}$$

W: $\Delta E \sim M_W c^2 \Rightarrow R_W \sim \frac{197 \text{ MeV fm}}{80.4 \text{ GeV}} = \underline{\underline{2.5 \cdot 10^{-3} \text{ fm}}}$

Z: $\Delta E \sim M_Z c^2 \Rightarrow R_Z \sim \frac{197 \text{ MeV fm}}{91.2} = \underline{\underline{2.2 \cdot 10^{-3} \text{ fm}}}$

From PDG, charge radius of proton $R_p = 0.870 \pm 0.08 \text{ fm}$

$$\Rightarrow \left[\frac{R_{W/Z}}{R_p} \sim \frac{1}{400} \right]$$

(b) In the LAB, time dilation $\tau \rightarrow \gamma \tau$

Distance $L = \gamma \tau \beta$ but $P = \gamma m \beta \Rightarrow L = \frac{P \tau}{m}$

If τ is in sec, multiply by c to get L in meters

$$\Rightarrow \left[L = \frac{P}{m} c \tau \right]$$

μ^+

$\tau = 2.2 \cdot 10^{-6} \text{ sec}$ WEAK

$m = 0.105 \text{ GeV} \Rightarrow L = \frac{10}{0.105} 2.2 \cdot 10^{-6} 3 \cdot 10^8 \text{ m} = \underline{\underline{63 \text{ km}}}$

τ^+

$\tau = 291 \cdot 10^{-15} \text{ sec}$
 $m = 1.78 \text{ GeV}$

WEAK
 $\Rightarrow L = \frac{10}{1.78} 291 \cdot 10^{-15} 3 \cdot 10^8 \text{ m} = \underline{\underline{490 \mu\text{m}}}$

K^+

$$\tau = 1.2 \cdot 10^{-8} \text{ sec} \quad \text{WEAK}$$

$$m = 0.494 \text{ GeV} \Rightarrow L = \frac{10}{0.494} \cdot 1.2 \cdot 10^{-8} \cdot 3 \cdot 10^8 \text{ m} = \underline{\underline{72 \text{ m}}}$$

 π^+

$$\tau = 2.6 \cdot 10^{-8} \text{ sec} \quad \text{WEAK}$$

$$m = 0.140 \text{ GeV} \Rightarrow L = \frac{10}{0.140} \cdot 2.6 \cdot 10^{-8} \cdot 3 \cdot 10^8 \text{ m} = \underline{\underline{560 \text{ m}}}$$

 π^0

$$\tau = 8.4 \cdot 10^{-17} \text{ sec} \quad \text{EM}$$

$$m = 0.135 \text{ GeV} \Rightarrow L = \frac{10}{0.135} \cdot 8.4 \cdot 10^{-17} \cdot 3 \cdot 10^8 \text{ m} = \underline{\underline{2 \mu\text{m}}}$$

 ρ^0

$$\Gamma = 149 \text{ MeV} \Rightarrow \tau = \frac{\hbar}{\Gamma} = \frac{6.6 \cdot 10^{-22} \text{ MeV sec}}{149 \text{ MeV}} = 4.4 \cdot 10^{-24} \text{ sec} \quad \text{STRONG}$$

$$m = 770 \text{ MeV} \Rightarrow L = \frac{10}{0.770} \cdot 4.4 \cdot 10^{-24} \cdot 3 \cdot 10^8 \text{ m} = \underline{\underline{17 \text{ fm}}}$$

 D^+

$$\tau = 1.1 \cdot 10^{-12} \text{ sec} \quad \text{WEAK}$$

$$m = 1.87 \text{ GeV} \Rightarrow L = \frac{10}{1.87} \cdot 1.1 \cdot 10^{-12} \cdot 3 \cdot 10^8 \text{ m} = \underline{\underline{1.8 \text{ mm}}}$$

 B^+

$$\tau = 1.7 \cdot 10^{-12} \text{ sec} \quad \text{WEAK}$$

$$m = 5.28 \text{ GeV} \Rightarrow L = \frac{10}{5.28} \cdot 1.7 \cdot 10^{-12} \cdot 3 \cdot 10^8 = \underline{\underline{0.9 \text{ mm}}}$$

 J/ψ

$$\Gamma = 87 \text{ keV} \Rightarrow \tau = \frac{\hbar}{\Gamma} = \frac{6.6 \cdot 10^{-22} \text{ MeV sec}}{87 \cdot 10^{-3} \text{ MeV}} = 7.6 \cdot 10^{-21} \text{ sec}$$

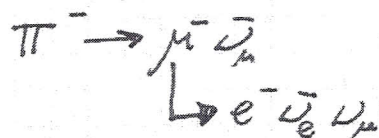
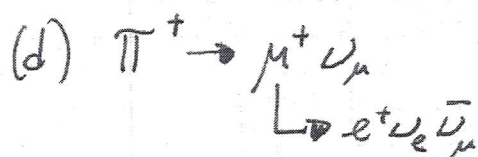
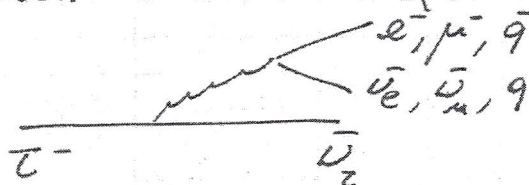
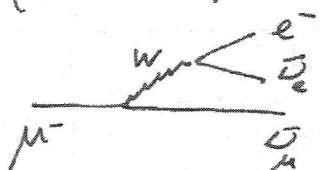
The lifetime of the J/ψ is a bit ~~longer~~ longer than a typical strong lifetime, but also a bit shorter than a typical ~~strong~~ EM lifetime -

The J/ψ can decay both STRONGLY or ELECTROMAGNETICALLY. The strong decays are suppressed (we'll understand why later in the course), and its strong decay rate is comparable to its EM -

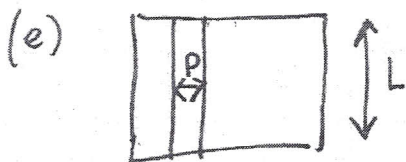
$$m = 3.1 \text{ GeV} \Rightarrow L = \frac{10}{3.1} 7.6 \cdot 10^{-21} 3 \cdot 10^8 = \underline{735 \text{ fm}}$$

- (c) The μ is lighter than the lightest hadron (π)
 $\Rightarrow$ can only decay into lighter charged leptons.
 (the only possibility is the electron)

The τ can ~~both~~ decay both into lighter leptons (electrons, muons) as well as hadrons (because $m_\tau > m_\pi$)



Therefore $\frac{(\nu_\mu^-)}{(\nu_e^-)} \sim 2$



$$L = 3 \text{ cm}$$

$$p = 50 \mu$$

$$\Rightarrow \# \text{ of channels} = \frac{3 \text{ cm}}{50 \mu} = 600 \text{ channels}$$

Need $600 \times 1000 = 6 \cdot 10^5$ cosmic rays going through detector
 Cosmic ray flux $\sim 1 \text{ cm}^{-2} \text{ min}^{-1}$ for horizontal detectors (PDG)
 Surface area $A = 9 \text{ cm}^2$

$$\text{Need to wait } \frac{6 \cdot 10^5}{9} \text{ min} = 6.7 \cdot 10^4 \text{ min} = 1.1 \cdot 10^3 \text{ hr} = \underline{46 \text{ days}}$$

(f) K^+ is $u\bar{s}$. The final state $e^+\bar{\nu}_e$ ($e^+\nu_e$) must be coupled to a W^- (W^+) in the weak decay of a quark.

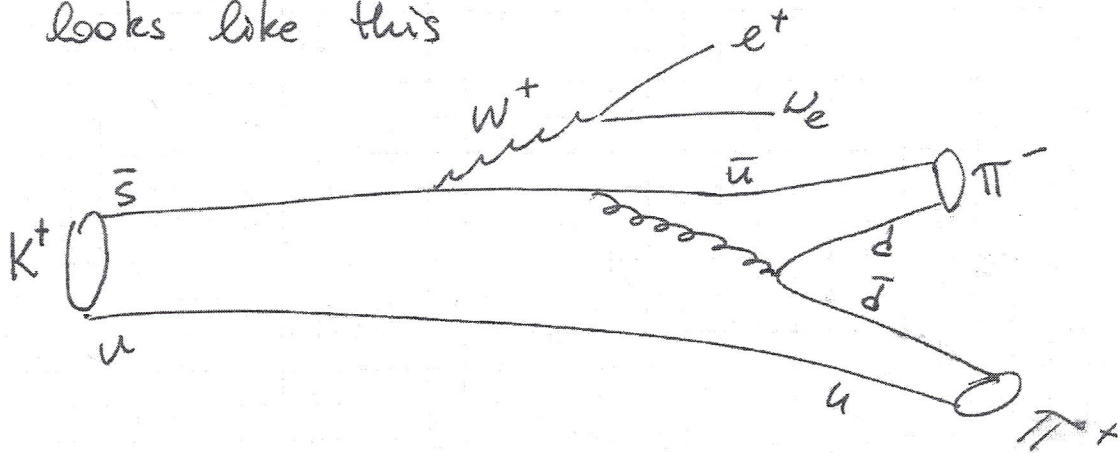
The quark that undergoes weak decay is the heaviest ($\bar{s}$). $\bar{s} (Q=+1/3) \rightarrow \bar{u} (Q=-2/3) W^+$

This is why the final state with a positron has been observed, $BR(K^+ \rightarrow \pi^+\pi^-\bar{e}^+\nu_e) \sim 4 \times 10^{-5}$, while the final state with an electron has never been seen, $BR(K^+ \rightarrow \pi^+\pi^+e^-\bar{\nu}_e) < 10^{-8}$.

This is known as the $\Delta S = \Delta Q$ rule.

(S is strangeness, Q is charge)

The Feynman diagram for the allowed decay looks like this



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$$(i) \quad y = \frac{1}{2} \log \frac{E + P_L}{E - P_L}$$

$$\frac{d^3 \vec{p}}{E} = \frac{dP_T^2 dP_L d\phi}{2E}$$

(like the volume element
in cylindrical coordinates)

$$dV = \frac{dp^2 dz d\phi}{2} \quad \uparrow \quad \phi = \text{azimuthal angle}$$

$$\text{Let } z = \frac{P_L}{E} \quad y = \frac{1}{2} \log \frac{1+z}{1-z}$$

$$\frac{dy}{dz} = \frac{1}{2} \frac{(1-z)}{(1+z)} \frac{(1+z) + (1-z)}{(1-z)^2} = \frac{1}{1-z^2}$$

$$\text{Also, } z = \frac{P_L}{E} = \frac{P_L}{\sqrt{P_L^2 + P_T^2 + m^2}}$$

$$\frac{dz}{dP_L} = \frac{E - \frac{1}{2E} 2P_L^2}{E^2} = \frac{1-z^2}{E}$$

$$\text{Then } \frac{dy}{dP_L} = \frac{dy}{dz} \frac{dz}{dP_L} = \frac{1}{1-z^2} \frac{1-z^2}{E} = \frac{1}{E}$$

$$\text{So } \frac{dP_L}{E} = dy \Rightarrow \boxed{\frac{d^3 \vec{p}}{E} = \frac{dP_T^2 dy d\phi}{2}}$$

(ii) Longitudinal boost (γ, β)

$$P_L \rightarrow P_L' = \gamma (P_L + \beta E)$$

$$E \rightarrow E' = \gamma (E + \beta P_L)$$

$$P_T \rightarrow P_T' = P_T$$

$$y \rightarrow y' = ??$$

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$$y' = \frac{1}{2} \log \frac{E' + P_L'}{E' - P_L'} = \frac{1}{2} \log \frac{(E' + P_L')^2}{(E'^2 - P_L'^2)} = \frac{1}{2} \log \frac{(E' + P_L')^2}{P_T'^2 + m^2}$$

(P_T = transverse momentum)

$$y' = \log \frac{E' + P_L'}{\sqrt{P_T'^2 + m^2}} = \log \frac{\gamma E + \gamma \beta P_L + \gamma P_L + \gamma \beta E}{\sqrt{P_T^2 + m^2}}$$

$$y' = \log \frac{(E + P_L)(\gamma + \gamma \beta)}{\sqrt{P_T^2 + m^2}} = \underbrace{\log \frac{E + P_L}{\sqrt{P_T^2 + m^2}}}_{= y} + \log \gamma(1 + \beta)$$

$$\boxed{y' = y + \Delta y}$$

$$\Delta y = \log \gamma(1 + \beta) = \frac{1}{2} \log \frac{(1 + \beta)^2}{1 - \beta^2} = \frac{1}{2} \log \frac{1 + \beta}{1 - \beta}$$

$$e^{2\Delta y} = \frac{1 + \beta}{1 - \beta} \Rightarrow \boxed{\beta = \frac{e^{\Delta y} - e^{-\Delta y}}{e^{\Delta y} + e^{-\Delta y}} = \tanh \Delta y}$$

$$(iii) \quad y = \frac{1}{2} \log \frac{E + P_L}{E - P_L} = \log \frac{E + P_L}{\sqrt{P_T^2 + m^2}}$$

$y_{\max} = -y_{\min}$ obtained by $P_L \rightarrow -P_L$

At a given energy, $y_{\max}$ obtained for $P_T = 0$

$$\Rightarrow y_{\text{MAX}} = \log \frac{(E + P_L)_{\text{MAX}}}{m}$$

Need to conserve E and P . Consider a single particle with zero P_T recoiling against the rest of the collision products (denoted by X)



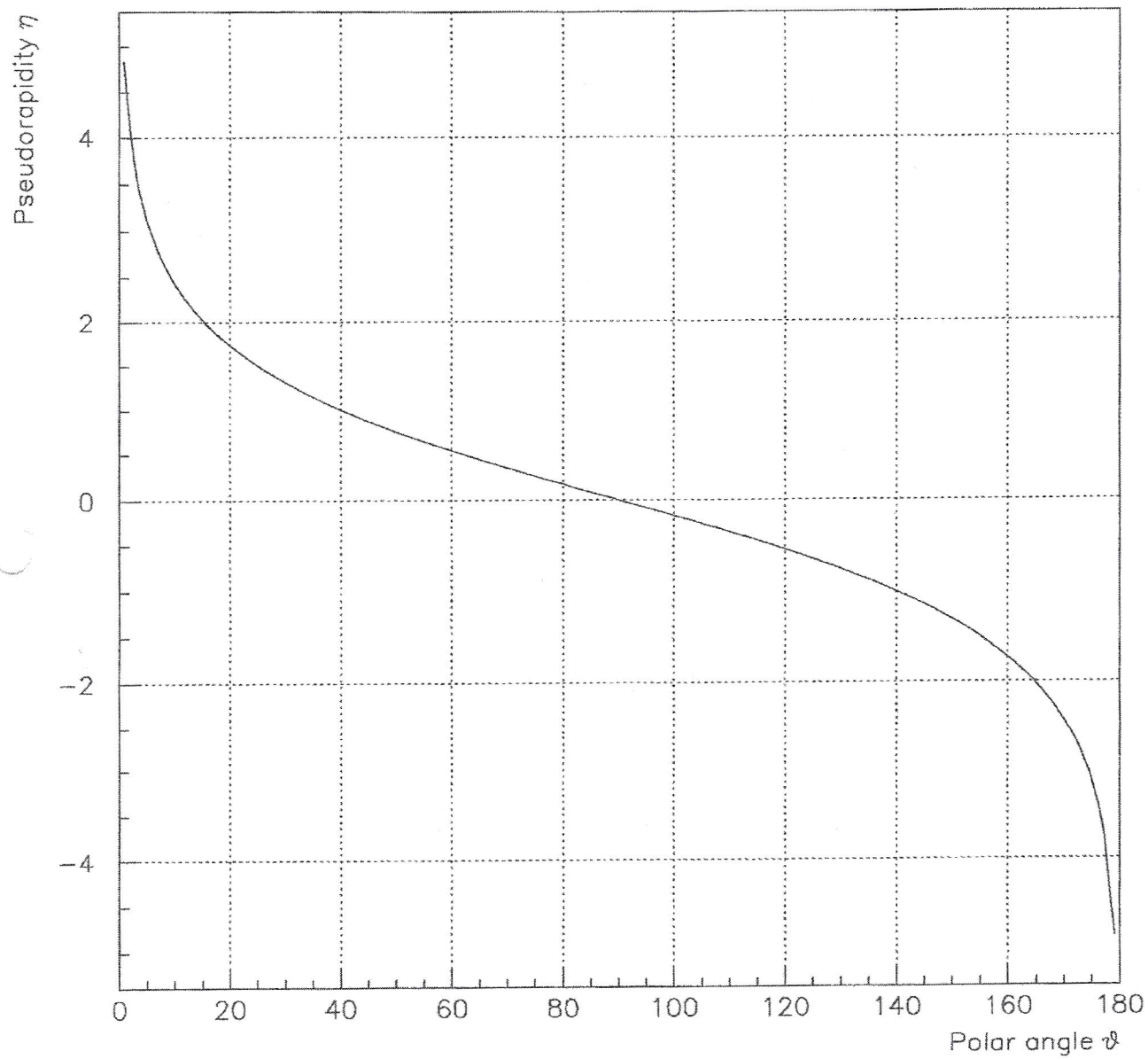
$$\left. \begin{aligned} E + E_x &= \sqrt{s} \\ E_x &= \sqrt{P_x^2 + m_x^2} \end{aligned} \right\} E + \sqrt{P_L^2 + m^2} = \sqrt{s}$$

$(E + P_L)_{\text{MAX}}$ occurs when $m_x = 0$

Then $E + P_L = \sqrt{s}$

$$\Rightarrow y_{\text{MAX}} = \log \frac{\sqrt{s}}{m} = \frac{1}{2} \log \frac{s}{m^2}$$

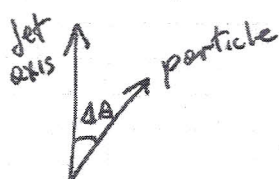
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- (v) Because
- ϕ is invariant under longitudinal boost
 - P_T is invariant under longitudinal boost
 - $y \rightarrow y + \Delta y$ under longitudinal boost
and $y \sim \eta$ -----

I will just show that jets are circular in $(\eta-\phi)$ space when they are emitted at 90° . Then, by the arguments above

- (1) jets will be circular in $(\eta-\phi)$ space independent of y (or θ)
- (2) the size of a jet of a given P_T will be independent of the θ of the jet



$$\Delta\theta \sim \frac{q_t}{q_e} \quad \Delta\phi \sim \frac{q_t}{q_e \sin\theta} = \frac{q_t}{q_e} \quad \text{at } \theta = 90^\circ$$

$$\Delta\eta = \Delta\theta \cdot \frac{d\eta}{d\theta} \quad \Delta\theta = \Delta\phi \text{ at } \theta = 90^\circ$$

$$\eta = \log \cot \frac{\theta}{2} \quad \frac{d\eta}{d\theta} = \tan \frac{\theta}{2} \times \frac{d}{d\theta} \frac{\cos \theta/2}{\sin \theta/2}$$

$$\frac{d\eta}{d\theta} = \tan \frac{\theta}{2} - \frac{\sin^2 \theta/2 - \cos^2 \theta/2}{\sin^2 \theta/2} \times \frac{1}{2}$$

$$\frac{d\eta}{d\theta} = \frac{\tan \theta/2}{2} \times \frac{(-1)}{\sin^2 \theta/2}$$

$$\frac{d\eta}{d\theta} = \frac{1 - \cos \theta}{2} (-1) \frac{2}{1 - \cos \theta} = -1$$

We had $\Delta\eta = \Delta\theta \frac{d\eta}{d\theta} \Rightarrow \Delta\eta = \Delta\theta$ (dropping the
meaningless
minus sign!)

But $\Delta\theta = \Delta\phi$ at $\theta = 90^\circ$

$$\Rightarrow \text{At } \theta = 90^\circ, \boxed{\Delta\eta = \Delta\phi}$$

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Let $f(\theta_y) d\theta_y$ be the probability per unit length
 (i) that the particle is scattered through an angle θ_y
 in the (yz) plane

$$\langle \theta_y \rangle = \int_{-\infty}^{\infty} \theta_y f(\theta_y) d\theta_y = 0 \quad \text{because } f(\theta_y) = f(-\theta_y)$$

$$\langle \theta_y^2 \rangle = \int_{-\infty}^{\infty} \theta_y^2 f(\theta_y) d\theta_y = \theta_0^2 \quad \text{by definition}$$

~

Consider the change in N going from $z \rightarrow z + dz$

$$\begin{aligned} N(z+dz, y, \theta_y) = & N(z, y - \theta_y dz, \theta_y) + \\ & + \int N(z, y, \theta_y - \theta'_y) f(\theta'_y) d\theta'_y dz + \\ & - \int N(z, y, \theta_y) f(\theta'_y) d\theta'_y dz \end{aligned}$$

where

- the 1st term is due to the drift, i.e. particles whose displacement changed because $\theta_y \neq 0$ (in general) -
- the 2nd term represents particles ^{that} ~~which~~ had the "right" y but the "wrong" θ_y and then scattered into the ~~correct~~ "right" θ_y bin
- the 3rd term represents particles ~~that~~ that had

the "right" y and θ_y , but, due to a further scatter ended up having the "wrong" θ_y

Now let us expand in Taylor series, keeping only ~~the~~ leading non-zero terms. Assume that in 2nd and 3rd term contributions are important only when θ_y' is small. Then

$$\begin{aligned} N(z, y, \theta_y) + \frac{\partial N}{\partial z} dz &= N(z, y, \theta_y) - \frac{\partial N}{\partial y} \theta_y dz + \\ &+ \int N(z, y, \theta_y) f(\theta_y') d\theta_y' dz + \\ &- \int \frac{\partial N}{\partial \theta_y} \theta_y' f(\theta_y') d\theta_y' dz + \\ &+ \int \frac{1}{2} \frac{\partial^2 N}{\partial \theta_y^2} \theta_y'^2 f(\theta_y') d\theta_y' dz + \\ &- \int N(z, y, \theta_y) f(\theta_y') d\theta_y' dz \end{aligned}$$

$$\begin{aligned} \frac{\partial N}{\partial z} &= -\frac{\partial N}{\partial y} \theta_y - \frac{\partial N}{\partial \theta_y} \underbrace{\int \theta_y' f(\theta_y') d\theta_y'}_{= \langle \theta_y \rangle = 0} + \frac{1}{2} \frac{\partial^2 N}{\partial \theta_y^2} \underbrace{\int \theta_y'^2 f(\theta_y') d\theta_y'}_{= \langle \theta_y^2 \rangle = \theta_0^2} \end{aligned}$$

$$\boxed{\frac{\partial N}{\partial z} = -\theta_y \frac{\partial N}{\partial y} + \frac{1}{2} \theta_0^2 \frac{\partial^2 N}{\partial \theta_y^2}}$$



$$(ii) \text{ Let } [\dots] = -\frac{2}{\theta_0^2} \left(\frac{\theta^2}{z} - \frac{3y\theta}{z^2} + \frac{3y^2}{z^3} \right)$$

$$\frac{\partial N}{\partial z} = -\frac{2\sqrt{3}N_0}{\pi\theta_0^2} \frac{1}{z^3} \exp[\dots] + \frac{4\sqrt{3}N_0}{\pi\theta_0^2} \frac{1}{z^2} \left(-\frac{\theta^2}{z^2} + \frac{6y\theta}{z^3} - \frac{9y^2}{z^4} \right) \times \exp[\dots]$$

$$\frac{\partial N}{\partial z} = \frac{2\sqrt{3}N_0}{\pi\theta_0^2} \exp[\dots] \cdot \left\{ -\frac{1}{z^3} + \frac{\theta^2}{z^4\theta_0^2} - \frac{6y\theta}{z^5\theta_0^2} - \frac{9y^2}{z^6\theta_0^2} \right\} \quad [1]$$

$$\frac{\partial N}{\partial y} = \frac{4\sqrt{3}N_0}{\pi\theta_0^2} \frac{1}{z^2} \left(-\frac{2}{\theta_0^2} \right) \left(-\frac{3\theta}{z^2} + \frac{6y}{z^3} \right) \exp[\dots]$$

$$\frac{\partial N}{\partial y} = \frac{2\sqrt{3}N_0}{\pi\theta_0^2} \exp[\dots] \left\{ +\frac{3\theta}{z^4\theta_0^2} - \frac{6y}{z^5\theta_0^2} \right\}$$

$$-\theta \frac{\partial N}{\partial y} = \frac{2\sqrt{3}N_0}{\pi\theta_0^2} \exp[\dots] \left\{ -\frac{3\theta}{z^4\theta_0^2} + \frac{6y\theta}{z^5\theta_0^2} \right\} \quad [2]$$

$$\frac{\partial N}{\partial \theta} = \frac{4\sqrt{3}N_0}{\pi\theta_0^2} \frac{1}{z^2} \left[-\frac{2}{\theta_0^2} \right] \left[\frac{2\theta}{z} - \frac{3y}{z^2} \right] \exp[\dots]$$

$$\frac{\partial N}{\partial \theta} = \frac{2\sqrt{3}N_0}{\pi\theta_0^2} \left[-\frac{2\theta}{\theta_0^2 z^3} + \frac{3y}{z^4\theta_0^2} \right] \exp[\dots]$$

$$\frac{\partial N}{\partial \theta} = \frac{2\sqrt{3}N_0}{\pi\theta_0^2} \left[-\frac{2}{\theta_0^2 z^3} \right] \exp[\dots] + \frac{8\sqrt{3}N_0}{\pi\theta_0^2} \left[-\frac{2\theta}{\theta_0^2 z^3} + \frac{3y}{z^4\theta_0^2} \right] \times \left(-\frac{2}{\theta_0^2} \right) \left(\frac{2\theta}{z} - \frac{3y}{z^2} \right) \exp[\dots]$$

$$\frac{\partial N}{\partial \theta} = \frac{2\sqrt{3}N_0}{\pi\theta_0^2} \left[-\frac{2}{z^3\theta_0^2} + \frac{8\theta}{\theta_0^4 z^4} - \frac{12\theta y}{\theta_0^4 z^5} - \frac{12\theta y}{\theta_0^4 z^5} - \frac{18y^2}{z^6\theta_0^6} \right] \times \exp[\dots]$$

$$\frac{A_0^2}{2} \frac{\partial^2 N}{\partial A_y^2} = \frac{2\sqrt{3}N_0}{\pi A_0^2} \left[-\frac{1}{z^3} + \frac{4A_y^2}{\theta_0^2 z^3} - \frac{12A_y y}{A_0^2 z^5} - \frac{9y^2}{\theta_0^2 z^6} \right] \exp[\dots]$$

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We need to verify that $\boxed{1} = \boxed{2} + \boxed{3}$

The terms that multiply $\frac{2\sqrt{3}N_0}{\pi A_0^2} \exp[\dots]$ are

$$\boxed{1}: \quad -\frac{1}{z^3} + \frac{A_y^2}{z^4 A_0^2} - \frac{6y A_y}{z^5 \theta_0^2} - \frac{9y^2}{z^6 \theta_0^2}$$

$$\boxed{2}: \quad -\frac{3\theta_y^2}{z^4 A_0^2} + \frac{6y A_y}{z^5 A_0^2}$$

$$\boxed{3}: \quad -\frac{1}{z^3} + \frac{4A_y^2}{z^4 A_0^2} - \frac{12y A_y}{z^5 \theta_0^2} - \frac{9y^2}{z^6 \theta_0^2}$$

Yes!

$$\boxed{1} = \boxed{2} + \boxed{3}$$

$$(iii) N(z, A_y) = \frac{\sqrt{3} N_0}{\pi \theta_0^2 z^2} \exp\left[-\frac{2A_y^2}{\theta_0^2 z}\right] dA_y \int_{-\infty}^{\infty} \exp\left[\frac{6yA_y}{\theta_0^2 z^2} - \frac{6y^2}{\theta_0^2 z^3}\right] dy$$

CALL THIS A

Use $\int_{-\infty}^{\infty} e^{-(ax^2+bx)} dx = \sqrt{\frac{\pi}{a}} e^{b^2/4a}$

In our case $a = \frac{6}{\theta_0^2 z^3}$ $b = -\frac{6A_y}{\theta_0^2 z^2}$, so

$$N(z, A_y) = A \sqrt{\frac{\pi \theta_0^2 z^3}{6}} \exp\left(\frac{36 A_y^2}{\theta_0^4 z^4} \cdot \frac{1}{4} \frac{\theta_0^2 z^3}{6}\right)$$

$$N(z, A_y) = \frac{\sqrt{3} N_0}{\pi \theta_0^2 z^2} \frac{\sqrt{\pi} \theta_0 z \sqrt{z}}{\sqrt{3} \sqrt{2}} \exp\left[-\frac{2A_y^2}{\theta_0^2 z} + \frac{3\theta_0^2 z}{2\theta_0^2 z}\right]$$

$$N(z, A_y) = \frac{N_0}{\sqrt{2\pi} \theta_0 \sqrt{z}} \exp$$

$$N(z, A_y) dA_y = \frac{N_0}{\sqrt{2\pi} \theta_0 \sqrt{z}} \exp\left[-\frac{A_y^2}{2\theta_0^2 z}\right]$$

Gaussian, normalized to 1, mean μ , rms σ is

$$G(x; \mu; \sigma) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Identify $\boxed{\mu=0}$ $\boxed{\sigma = \theta_0 \sqrt{z}}$

and the normalization is correct, i.e. total number of particles is N_0

~~Actually it is wrong - it is normalized to 1~~



(iv)

Dropping the normalization now

$$N(z, y) dy \sim \exp - \frac{6y^2}{\theta_0^2 z^3} \int_{-\infty}^{\infty} \exp \left[-\frac{2}{\theta_0^2} \left(\frac{\theta_y^2}{z} - \frac{3y\theta_y}{z^2} \right) \right] d\theta_y dy$$

Again, use $\int_{-\infty}^{\infty} e^{-(ax^2+bx)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{b^2/4a}$

In this case $a = \frac{2}{\theta_0^2 z}$ $b = -\frac{6y}{\theta_0^2 z^2}$, so

$$N(z, y) dy \approx \exp - \frac{6y^2}{\theta_0^2 z^3} \cdot \exp \frac{36y^2}{\theta_0^4 z^4} \frac{\theta_0^2 z}{8}$$

$$N(z, y) dy \sim \exp - \frac{3y^2}{2\theta_0^2 z^3}$$

Gaussian $\sim \exp \left[-\frac{(y-\mu)^2}{2\sigma^2} \right] \Rightarrow \mu=0$

$$\sigma^2 = \frac{\theta_0^2 z^3}{3}$$

$$\sigma = \frac{\theta_0 z^{3/2}}{\sqrt{3}}$$

(v) At $z=t$, $y=d$ ~~$N(y, \theta_y) dy d\theta_y$~~ Dropping the normalization

$$N(\theta_y) d\theta_y \sim \exp \left[-\frac{2}{\theta_0^2} \left(\frac{\theta_y^2}{t} - \frac{3d\theta_y}{t^2} \right) \right] d\theta_y$$

$$\sim \exp \left[-\left(\frac{2\theta_y^2}{t\theta_0^2} - \frac{6d\theta_y}{t^2\theta_0^2} \right) \right] d\theta_y$$

$$\sim \exp \left[-\left(\frac{2\theta_y^2}{t\theta_0^2} - \frac{6d\theta_y}{t^2\theta_0^2} + \frac{9d^2}{2t^3\theta_0^2} - \frac{9d^2}{2t^3\theta_0^2} \right) \right] d\theta_y$$

(we are completing the square in the exponent)

$$N(\theta_y) d\theta_y \sim \exp \left[- \left(\frac{\sqrt{2} \theta_y}{\theta_0 \sqrt{t}} - \frac{3d}{\sqrt{2} t^{3/2} \theta_0} \right)^2 \right]$$

$$N(\theta_y) d\theta_y \sim \exp \left[- \left(\frac{2\theta_y t - 3d}{\sqrt{2} t^{3/2} \theta_0} \right)^2 \right]$$

$$N(\theta_y) d\theta_y \sim \exp - \frac{4t^2 (\theta_y - \frac{3}{2} d/t)^2}{2t^3 \theta_0^2}$$

$$N(\theta_y) d\theta_y \sim \exp - \frac{2 (\theta_y - \frac{3}{2} d/t)^2}{t \theta_0^2}$$

This is gaussian with $\boxed{\mu = \frac{3}{2} d/t}$

$$2\sigma^2 = \frac{t \theta_0^2}{2} \quad \boxed{\sigma = \frac{\theta_0 \sqrt{t}}{2}}$$

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(a) Bethe-Bloch, from PDG

$$\frac{dE}{dx} = -K \underset{\substack{\uparrow \\ =1}}{Z^2} \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \log \frac{2m_e c^2 \beta^2 \gamma^2 T_{MAX}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

$$T_{MAX} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma(m_e/M) + (m_e/M)^2}$$

M = mass of projectile

↑ ignore for gas

Express everything in terms of momentum, or p/M

$$\gamma\beta = \frac{p}{M} = \eta$$

$$\frac{\beta^2}{1-\beta^2} = \eta^2 \rightarrow \beta^2 = \frac{\eta^2}{\eta^2+1}$$

$$\gamma^2 = \frac{1}{1-\beta^2} = \frac{\eta^2}{\beta^2} = \eta^2+1 \rightarrow \gamma = \sqrt{\eta^2+1}$$

$$\frac{dE}{dx} = -\frac{KZ}{A} \frac{\eta^2+1}{\eta^2} \left[\frac{1}{2} \log \frac{2m_e \eta^2 T_{MAX}}{I^2} - \frac{\eta^2}{\eta^2+1} \right]$$

We are concerned with π, μ, K, p .

For these particles $M \gg m_e \Rightarrow T_{MAX} \sim 2m_e c^2 \beta^2 \gamma^2$

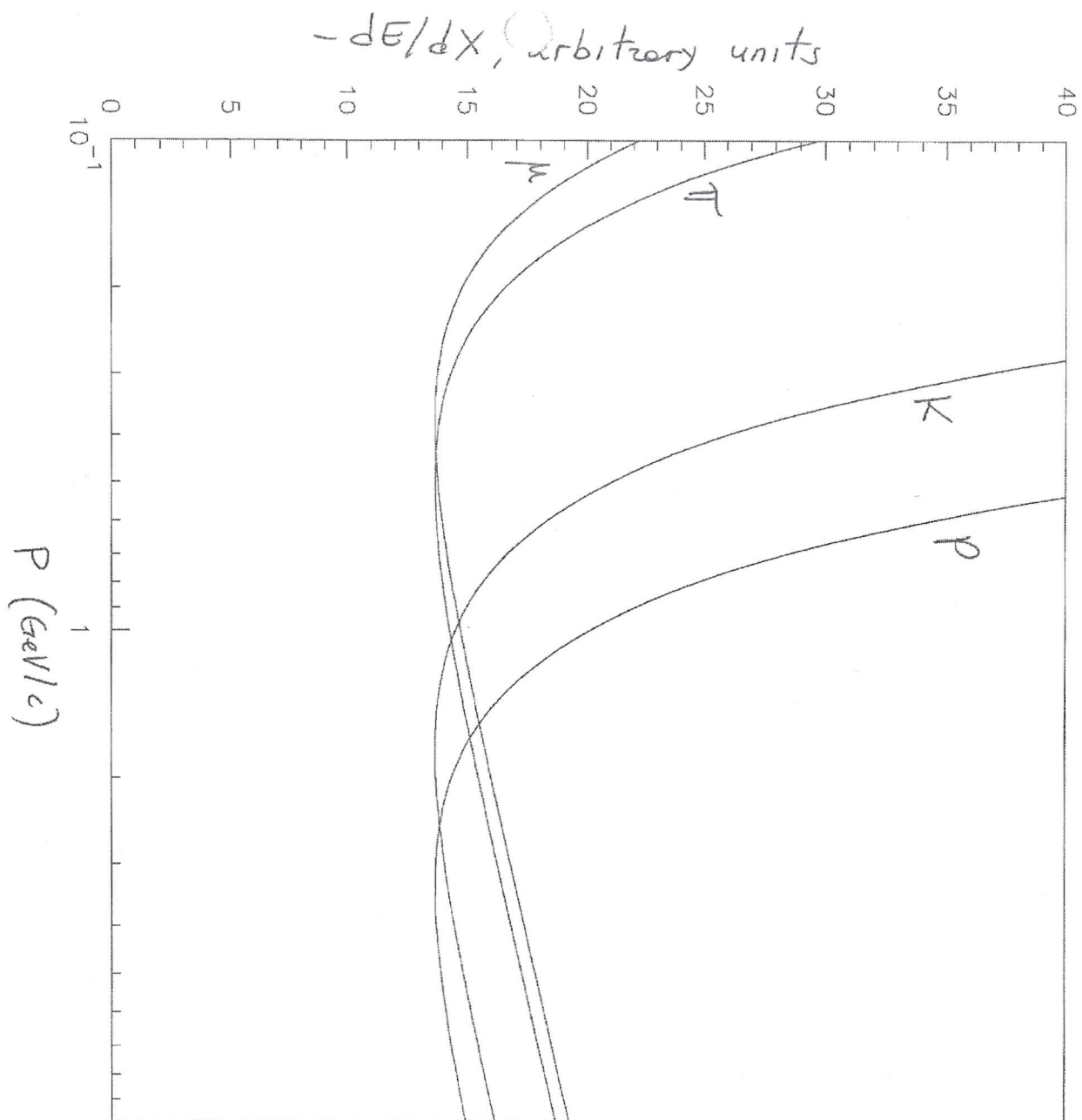
So

$$\frac{dE}{dx} = -\frac{KZ}{A} \frac{\eta^2+1}{\eta^2} \left[\frac{1}{2} \log \frac{4m_e^2 \eta^4}{I^2} - \frac{\eta^2}{\eta^2+1} \right]$$

Using $I = 16 \text{ eV}$, plot $\frac{dE}{dx}$ in arbitrary units

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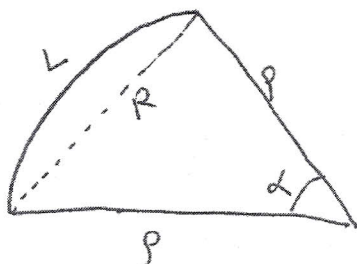
(b)

Ignoring the bend

$$t = \frac{R}{\beta c}$$

Again let $\eta = \frac{P}{M}$, then $\beta = \frac{\eta}{\sqrt{\eta^2 + 1}}$

$$t = \frac{\sqrt{\eta^2 + 1}}{\eta} \frac{R}{c}$$

with $R = 1.5 \text{ m}$ and $c = 3 \times 10^8 \text{ m/sec}$ Including the bend, the path-length is not R The radius of curvature ρ is given by $\rho = \frac{P}{0.3 B}$ where P is in GeV/c, ρ is in m, B is in Tesla

$$L = \rho \alpha$$

$$R^2 = \rho^2 + \rho^2 - 2\rho^2 \cos \alpha$$

$$R^2 = 2\rho^2 (1 - \cos \alpha)$$

$$R^2 = 2\rho^2 \sin^2 \frac{\alpha}{2}$$

$$\sin \frac{\alpha}{2} = \frac{R}{2\rho}$$

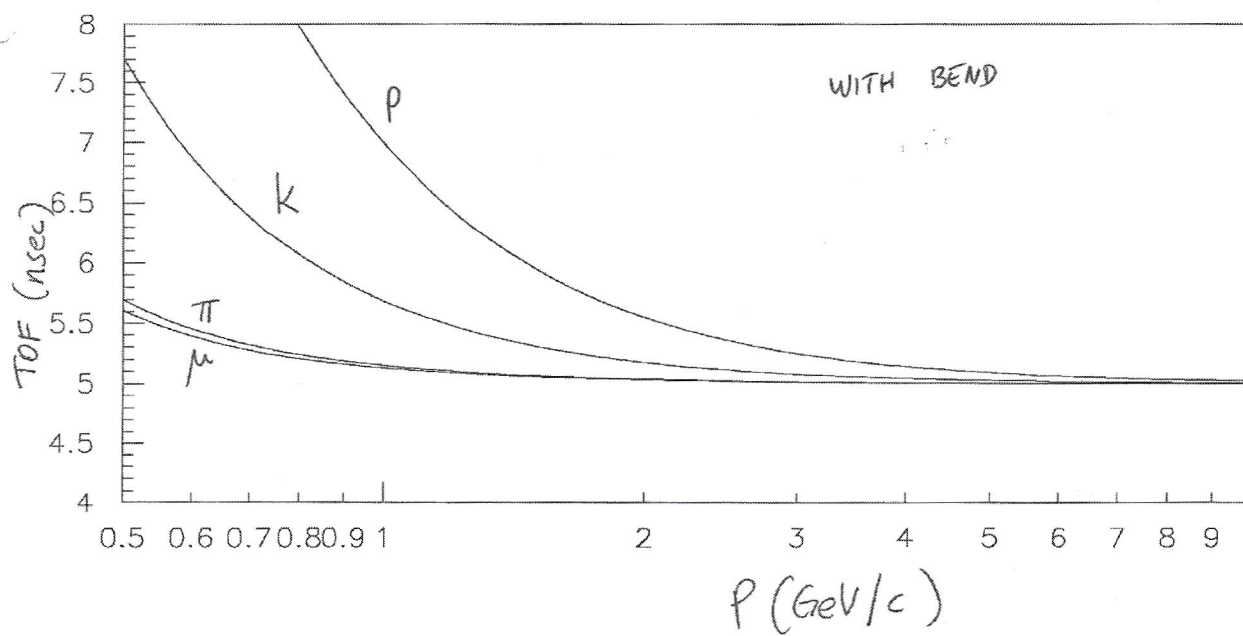
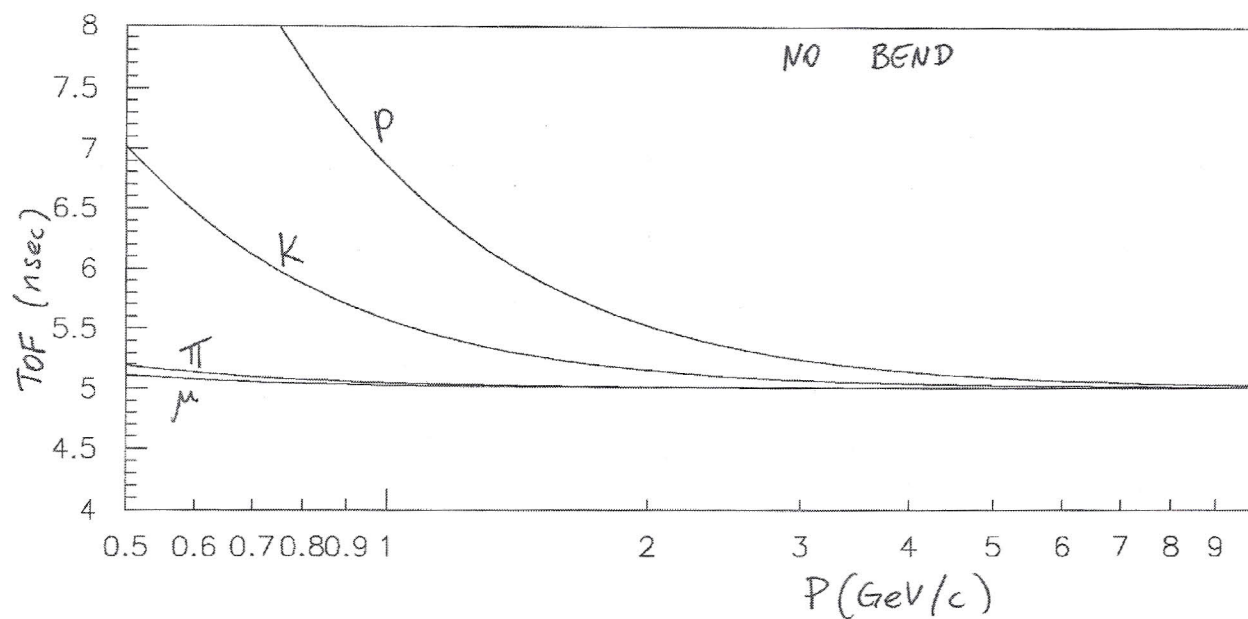
$$\alpha = 2 \sin^{-1} \frac{R}{2\rho}$$

$$\text{So } L = 2\rho \sin^{-1} \frac{R}{2\rho}, \text{ and } t = \frac{\sqrt{\eta^2 + 1}}{\eta} \frac{L}{c}$$

$$t = 2 \frac{\sqrt{\eta^2 + 1}}{\eta} \frac{\rho}{c} \sin^{-1} \frac{R}{2\rho}$$

$$\text{with } \rho = \frac{P}{0.3 B}$$

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$$\text{In Fe } \left. \frac{dE}{dx} \right|_{\text{MIN}} \sim 1.45 \text{ MeV } \frac{\text{cm}^2}{\text{g}}$$

$$\rho = 7.87 \frac{\text{g}}{\text{cm}^3} \Rightarrow \left. \frac{dE}{dx} \right|_{\text{MIN}} = 11.4 \frac{\text{MeV}}{\text{cm}}$$

$$\text{Kinetic Energy } T = E - m = \sqrt{p^2 + m^2} - m$$

In order to lose all the kinetic energy we must have a thickness

$$\Delta x \sim T / \left. \frac{dE}{dx} \right|_{\text{MIN}} = (\sqrt{p^2 + m^2} - m) \times \left. \frac{dE}{dx} \right|_{\text{MIN}}$$

$$P = 1 \text{ GeV} \Rightarrow \Delta x = \frac{(\sqrt{1 + (0.106)^2} - 0.106)}{11.4 \cdot 10^{-3}} \text{ cm}$$

$$\boxed{\Delta x \approx 80 \text{ cm}}$$

$$P = 5 \text{ GeV} \Rightarrow \Delta x = \frac{5 - 0.106}{11.4 \cdot 10^{-3}} \text{ cm} \quad \boxed{\Delta x = 430 \text{ cm}}$$

$$P = 10 \text{ GeV} \Rightarrow \Delta x = \frac{10 - 0.106}{11.4 \cdot 10^{-3}} \text{ cm} \quad \boxed{\Delta x = 870 \text{ cm}}$$

$$\text{Also } \lambda_I \approx 132 \frac{\text{g}}{\text{cm}^2} \rightarrow \lambda_I = \frac{132}{7.87} \text{ cm} \approx 17 \text{ cm}$$

$$\text{for } \Delta x = 80 \text{ cm} \quad \boxed{e^{-80/17} \sim 1\%} \leftarrow \text{about right}$$

$$\Delta x = 430 \text{ cm} \quad \boxed{e^{-430/17} \sim 10^{-11}} \leftarrow \text{unreasonably small}$$

$$\Delta x = 870 \text{ cm} \quad \boxed{e^{-870/17} \sim 10^{-22}} \leftarrow \text{Other effects come into play}$$